An Approximate Reflectance Profile for Efficient Subsurface Scattering

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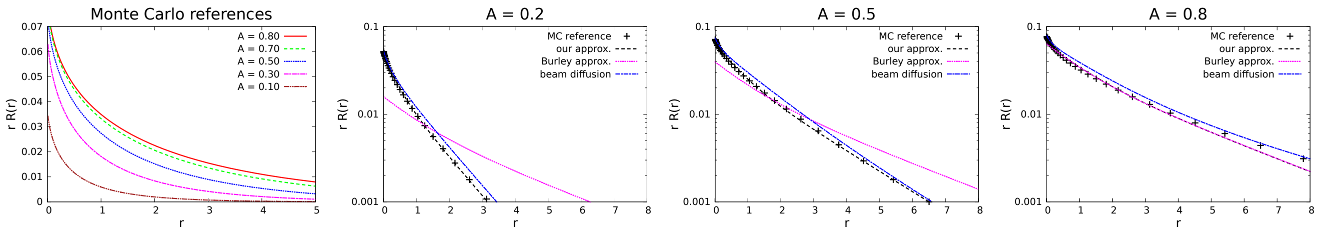


Figure 1: *(a) MC reference curves. (b)–(d) Fit of various reflectance profile models for surface albedos 0.2, 0.5, 0.8 (log vertical axes).*

1 Introduction

Computer graphics researchers have developed increasingly sophisticated and accurate physically-based subsurface scattering BSSRDF models: from the simple dipole diffusion model [Jensen et al. 2001] to the quantized diffusion [d'Eon and Irving 2011] and beam diffusion [Habel et al. 2013] models. We present a BSSRDF model based on an empirical reflectance profile that is as simple as the dipole but matches brute-force Monte Carlo references better than even beam diffusion.

Advantages of our empirical model: 1) no need to numerically invert the intuitive surface albedo A and mean free path length ℓ input parameters to volume scattering and absorption coefficients; 2) built-in single-scattering term; 3) faster and simpler evaluation.

2 Functional approximation

The BSSRDF S is often simplified as a product of a 1D diffuse *reflectance profile* R and directional Fresnel transmission terms F_t: $S(x_i, w_i; x_o, w_o) = C\, F_t(x_i, w_i)\, R(|x_o - x_i|)\, F_t(x_o, w_o)$. Figure 1(a) shows reflectance profiles for various surface albedos computed with brute-force MC particle tracing (mean free path $\ell = 1$, anisotropy $g = 0$). These are our reference curves.

Simple approximations of $R(r)$ with e.g. a cubic polynomial or a sum of Gaussians have been used for path-traced and point-based subsurface scattering. Burley [2013] noted that the shape of $R(r)$ can be approximated quite well with a sum of two exponential functions divided by distance r: $R(r) = \frac{e^{-r/\ell} + e^{-r/(3\ell)}}{8\,\pi\,\ell\,r}$. Here we analyze how to scale and stretch this function to match MC references for all possible surface albedos. We introduce albedo-dependent weight w and scale s of Burley's two exponentials:

$$R(r) = w\,\frac{e^{-sr/\ell} + e^{-sr/(3\ell)}}{8\,\pi\,\ell\,r}\;. \tag{1}$$

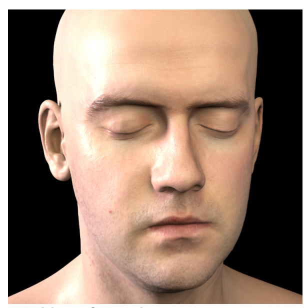
For curve fitting it is sufficient to consider $\ell = 1$ since the *shape* of the reference curves are independent of ℓ: $R(r, \ell) = R_{\ell=1}(\frac{r}{\ell}) \,/\, \ell^2$. Also, w has to equal $A\,s$ to make $\int_o^\infty R(r)\,2\pi r\,dr$ integrate to A.

The following simple expression for s gives a good fit to the MC references:

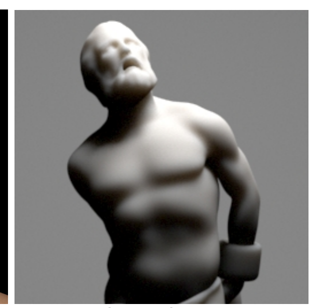
$$s = 1.85 - A + 7\,|A - 0.8|^3\;. \tag{2}$$

3 Results

Figures 1(b)–(d) show the fit of our approximation compared to MC references, Burley's approximation ($w = A$, $s = 1$), and beam diffusion. The relative error wrt. the references is on average 5.3% over the full range of albedos. Compared to all the approximations and assumptions implicitly built into the references (infinite plane, searchlight configuration, etc.) this is actually a modest error. The images below are rendered in RenderMan using our approximation.



Head data: Infinite Realities via Creative Commons



Prometheus statue by Scott Eaton

For importance sampling proportional to $R(r)r$ we can derive the corresponding cdf: $\mathrm{cdf}(r) = 1 - \frac{1}{4}e^{-sr/\ell} - \frac{3}{4}e^{-sr/(3\ell)}$. It is also possible to use a different parameterization of the scattering distance: diffuse mean free path on the surface, ℓ_d, instead of the mean free path in the volume, ℓ — this just requires a different expression for s. More details at: **graphics.pixar.com/library/ApproxBssrdf**.

References

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HABEL, R., CHRISTENSEN, P. H., AND JAROSZ, W. 2013. Photon beam diffusion: a hybrid Monte Carlo method for subsurface scattering. *EGSR*, 27–37.

JENSEN, H. W., MARSCHNER, S. R., LEVOY, M., AND HANRAHAN, P. 2001. A practical model for subsurface light transport. *SIGGRAPH*, 511–518.