

Approximate Reflectance Profiles for Efficient Subsurface Scattering



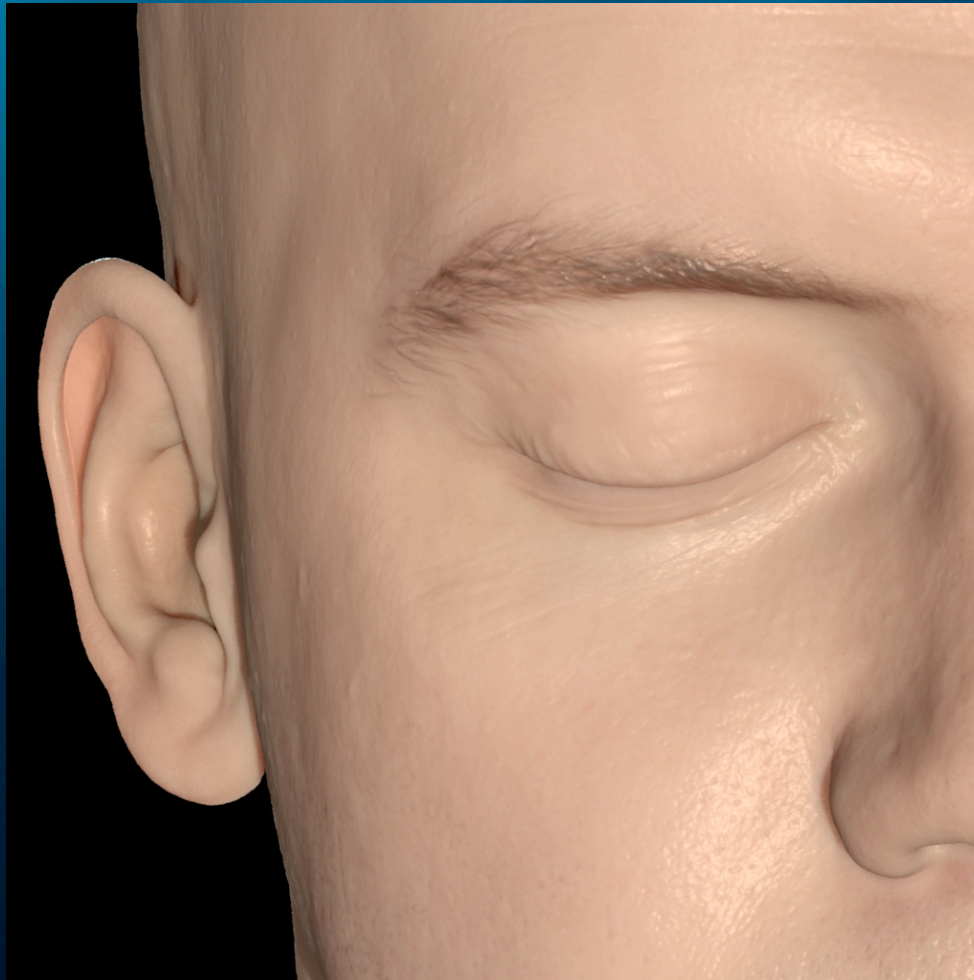
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SIGGRAPH 2015, Los Angeles

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Goal:
subsurface scattering, fast+simple



Overview

- Simple subsurface scattering model
- New parameterization allows comparison with physically-based models
- Matches Monte Carlo references very well -- better than physically-based models
- Useful for ray-traced (and point-based) subsurface scattering

Advantages

- Faster evaluation, simpler code
- Built-in single-scattering term
- No need for numerical inversion of user-friendly parameters (surface albedo and scattering length) to physical parameters (volume scattering and absorption coeffs)
- Bonus: simple cdf for importance sampling

Inspiration: Schlick's Fresnel approx.

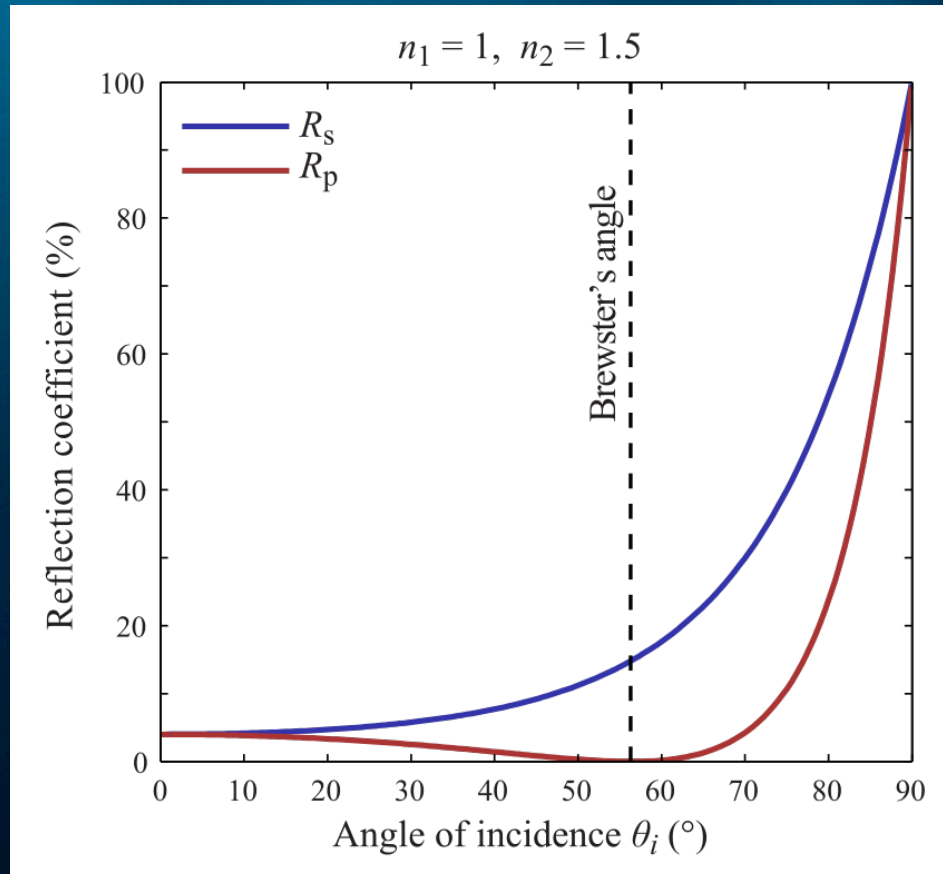
- Physics: Fresnel reflection formula -- reflection is average of parallel and perpendicular polarized:
 $R(\theta) = (R_p + R_s) / 2$

$$R_p = \left| \frac{n_1 \cos \theta_t - n_2 \cos \theta_i}{n_1 \cos \theta_t + n_2 \cos \theta_i} \right|^2 = \left| \frac{n_1 \sqrt{1 - \left(\frac{n_1}{n_2} \sin \theta_i\right)^2} - n_2 \cos \theta_i}{n_1 \sqrt{1 - \left(\frac{n_1}{n_2} \sin \theta_i\right)^2} + n_2 \cos \theta_i} \right|^2.$$

$$R_s = \left| \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} \right|^2 = \left| \frac{n_1 \cos \theta_i - n_2 \sqrt{1 - \left(\frac{n_1}{n_2} \sin \theta_i\right)^2}}{n_1 \cos \theta_i + n_2 \sqrt{1 - \left(\frac{n_1}{n_2} \sin \theta_i\right)^2}} \right|^2,$$

Inspiration: Schlick's Fresnel approx.

- Physics: Fresnel reflection formula



Inspiration: Schlick's Fresnel approx.

- [Schlick94]: Simple approximation as polynomial

$$R(\theta) = R_0 + (1 - R_0)(1 - \cos\theta)^5$$

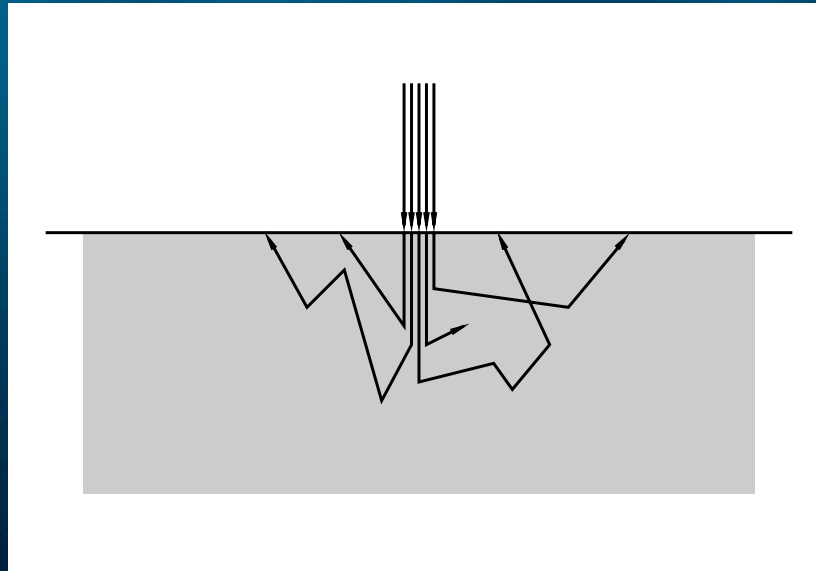
- No visual difference
- We want similar simple approximation for subsurface scattering!

Outline of talk

- Subsurface scattering
- Physically-based subsurface scattering models
- Burley's approximate model
- My reparameterization
- Results

Monte Carlo simulation

- Most general method: brute-force Monte Carlo



- But: very slow!

BSSRDF

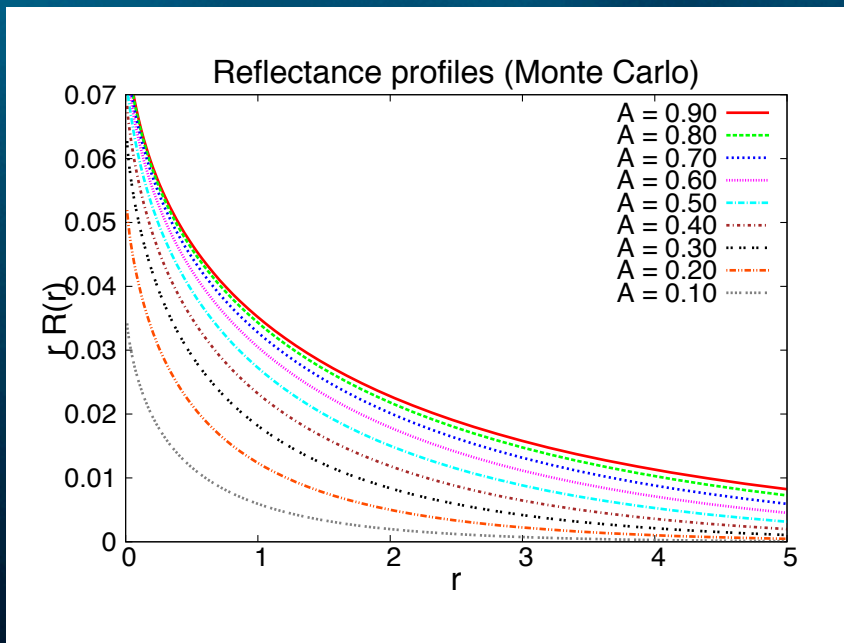
- Function that describes how light enters an object, bounces around, then leaves:
BSSRDF (bidirectional surface scattering reflectance distribution function) S
- Often simplified as:

$$S(x_i, w_i; x_o, w_o) = C F_t(x_i, w_i) R(|x_o - x_i|) F_t(x_o, w_o)$$

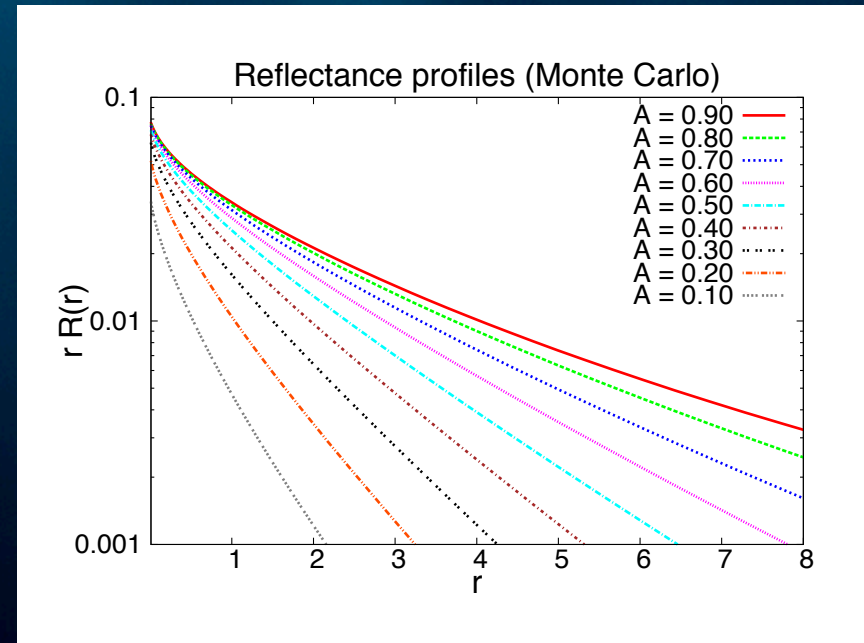
↑ reflectance profile
↑ Fresnel transmission terms

Reflectance profiles: reference

- Brute-force Monte Carlo simulation
- Reflectance profile $R(r)$; A = surface albedo



linear y axis



log y axis

Physically-based reflectance profiles

- Dipole diffusion [Jensen01,02]
 - simple, fast, widely used; but: blurry “waxy” look
- Better dipole diffusion [d’Eon12]
- Directional dipole diffusion [Frisvad14]
 - can handle oblique incident angles

Physically-based reflectance profiles

- Formulas:

Physically-based reflectance profiles

- Formulas:

$$R_d(r) = -D \frac{(\vec{n} \cdot \vec{\nabla} \phi(x_s))}{d\Phi_i}$$
$$= \frac{\alpha'}{4\pi} \left[(\sigma_{tr}d_r + 1) \frac{e^{-\sigma_{tr}d_r}}{\sigma'_t d_r^3} + z_v (\sigma_{tr}d_v + 1) \frac{e^{-\sigma_{tr}d_v}}{\sigma'_t d_v^3} \right].$$

Physically-based reflectance profiles

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$$= \frac{\alpha'}{4\pi} \left[(\sigma_{tr}d_r + 1) \frac{e^{-\sigma_{tr}d_r}}{\sigma_{tr}} + z_{\infty} (\sigma_{tr}d_{\infty} + 1) \frac{e^{-\sigma_{tr}d_{\infty}}}{\sigma_{tr}} \right]$$

$$R_d = 2\pi \int_0^{\infty} R_d(r) r dr = \frac{\alpha'}{2} \left(1 + e^{-\frac{4}{3}A\sqrt{3(1-\alpha')}} \right) e^{-\sqrt{3(1-\alpha')}}.$$

Physically-based reflectance profiles

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$$R_d = 2\pi \int_0^\infty R_d(r) r dr = \frac{\alpha'}{\omega} \left(1 + e^{-\frac{4}{3}A\sqrt{3(1-\alpha')}} \right) e^{-\sqrt{3(1-\alpha')}}.$$

$$L_o^{(1)}(x_o, \vec{\omega}_o) = \sigma_s(x_o) \int_{2\pi} F p(\vec{\omega}'_i \cdot \vec{\omega}'_o) \int_0^\infty e^{-\sigma_{tc}s} L_i(x_i, \vec{\omega}_i) ds d\vec{\omega}_i \quad (6)$$

$$= \int_A \int_{2\pi} S^{(1)}(x_i, \vec{\omega}_i; x_o, \vec{\omega}_o) L_i(x_i, \vec{\omega}_i) (\vec{n} \cdot \vec{\omega}_i) d\omega_i dA(x_i).$$

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$$\frac{(\alpha')^2}{4\pi} \left[\left(C_{\vec{E}} \frac{z_r (\mu_{tr} d_r + 1)}{d_r^2} + \frac{C_\phi}{D} \right) \frac{e^{-\mu_{tr} d_r}}{d_r} - \left(C_{\vec{E}} \frac{z_v (\mu_{tr} d_v + 1)}{d_v^2} + \frac{C_\phi}{D} \right) \frac{e^{-\mu_{tr} d_v}}{d_v} \right]$$

Physically-based reflectance profiles

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$$R_d = 2\pi \int_0^{\infty} R_d(r) r dr = \frac{\alpha'}{\alpha} \left(1 + e^{-\frac{4}{3}A\sqrt{3(1-\alpha')}} \right) e^{-\sqrt{3(1-\alpha')}}.$$

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$$\nabla \phi'_d = \frac{\Phi}{4\pi D} \frac{e^{-\sigma_{tr}r}}{r^3} \left(\vec{\omega}_{12} 3D(1 + \sigma_{tr}r) - \mathbf{x} (1 + \sigma_{tr}r) \left[\frac{d_v+1}{d_v^2} + \frac{C_\phi}{D} \right] \frac{e^{-\mu_{tr}d_v}}{d_v} \right)$$

$$- \mathbf{x} 3D \frac{3(1 + \sigma_{tr}r) + (\sigma_{tr}r)^2}{r} \cos \theta \Big). \quad (19)$$

Physically-based reflectance profiles

- Quantized diffusion [d'Eon11]
 - Improved diffusion theory
 - Extended source term (instead of just two points)
 - Sharper edges -- not “waxy” looking

Physically-based reflectance profiles

- More formulas:

Physically-based reflectance profiles

- More formulas:

$$\phi(r) = \frac{e^{-\mu_t r}}{4\pi r^2} + \frac{\mu_s}{2\pi^2 r} \int_0^\infty \frac{\arctan^2 u}{u - \alpha \arctan u} \sin(r \mu_t u) du.$$

Physically-based reflectance profiles

- More formulas:

$$\phi(r) = \frac{e^{-\mu_t r}}{4\pi r^2} + \frac{\mu_s}{2\pi^2 r} \int_0^\infty \frac{\arctan^2 u}{u - \alpha \arctan u} \sin(r \mu_t u) du.$$

$$\phi(r) = \frac{e^{-\mu'_t r}}{4\pi r^2} + \frac{1}{4\pi} \frac{3\mu'_s \mu'_t}{2\mu_a + \mu'_s} \frac{e^{-\sqrt{\frac{\mu_a}{D}} r}}{r}$$

Physically-based reflectance profiles

- More formulas:

$$\phi(r) = \frac{e^{-\mu_t r}}{4\pi r^2} + \frac{\mu_s}{2\pi^2 r} \int_0^\infty \frac{\arctan^2 u}{u - \alpha \arctan u} \sin(r \mu_t u) du.$$

$$\phi(r) = \frac{e^{-\mu_t' r}}{r} + \frac{1}{2\mu_a + \mu_s'} \frac{3\mu_s' \mu_t'}{r} e^{-\sqrt{\frac{\mu_a}{D}} r}$$

$$\int_0^\infty G_{3D}(v, \sqrt{r^2 + z^2}) Q(z) dz = \frac{1}{2} \mu_s' f(\mu_t'^2 v/2) G_{2D}(v, r),$$

Physically-based reflectance profiles

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$$\frac{1}{4\pi D} \frac{e^{-r\sqrt{\frac{\mu_a}{D}}}}{r} = \int_0^\infty \frac{c}{(4\pi Dct)^{3/2}} e^{-\mu_a ct} e^{-r^2/(4Dct)} dt$$

Physically-based reflectance profiles

- More formulas:

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$$\int_0^\infty G_{3D}(v, \sqrt{r^2 + z^2}) Q(z) dz = \frac{1}{2} \mu_s' f(\mu_t'^2 v/2) G_{2D}(v, r),$$

$$\int_{z_1}^{z_2} Q(z) G_{3D}(v, \sqrt{r^2 + (z+m)^2}) dz = G_{2D}(v, r) w_\phi(v, z_1, z_2, m),$$

$$w_\phi(v, z_1, z_2, m) = \int_{z_1}^{z_2} \frac{e^{-\frac{(-z+m)^2}{2v}}}{\sqrt{2\pi v}} \alpha' \mu_t e^{-\mu_t z} dz =$$

$$\frac{\alpha' \mu_t}{2} e^{m \mu_t + \frac{\mu_t^2 v}{2}} \left(\operatorname{erf} \left[\frac{m + \mu_t v + z_2}{\sqrt{2v}} \right] - \operatorname{erf} \left[\frac{m + \mu_t v + z_1}{\sqrt{2v}} \right] \right).$$

Physically-based reflectance profiles

- Photon beam diffusion [Habel13]
 - As accurate as quantized diffusion, but faster
 - Accurate single-scattering model
 - Can handle oblique incident angles

Physically-based reflectance profiles

- Some other formulas:

Physically-based reflectance profiles

- Some other formulas:

$$R_d \approx \frac{1}{N} \sum_{i=1}^N \frac{f(\vec{x}, \vec{\omega}, t_i) w_{\text{exp}}(t_i, \mathbf{x})}{w_{\text{exp}}(t_i, \mathbf{x}) \text{pdf}_{\text{exp}}(t_i) + w_{\text{eq}}(t_i, \mathbf{x}) \text{pdf}_{\text{eq}}(t_i | \vec{x}, \vec{\omega})} + \frac{1}{N} \sum_{j=1}^N \frac{f(\vec{x}, \vec{\omega}, t_j) w_{\text{eq}}(t_j, \mathbf{x})}{w_{\text{exp}}(t_j, \mathbf{x}) \text{pdf}_{\text{exp}}(t_j) + w_{\text{eq}}(t_j, \mathbf{x}) \text{pdf}_{\text{eq}}(t_j | \vec{x}, \vec{\omega})},$$

Physically-based reflectance profiles

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$$+ \frac{1}{N} \sum_{j=1}^N \frac{f(\vec{x}, \vec{\omega}, t_j) w_{\text{eq}}(t_j, \mathbf{x})}{w_{\text{exp}}(t_j, \mathbf{x}) \text{pdf}_{\text{eq}}(t_j | \vec{x}, \vec{\omega})}$$

$$R_{\vec{E}}^d(\vec{x}, t) = C_{\vec{E}} \frac{\alpha'}{4\pi} \left[\frac{z_r(t) (1 + \sigma_{tr} d_r(t)) e^{-\sigma_{tr} d_r(t)}}{d_r^3(t)} + \frac{(z_r(t) + 2z_b) (1 + \sigma_{tr} d_v(t)) e^{-\sigma_{tr} d_v(t)}}{d_v^3(t)} \right],$$

Physically-based reflectance profiles

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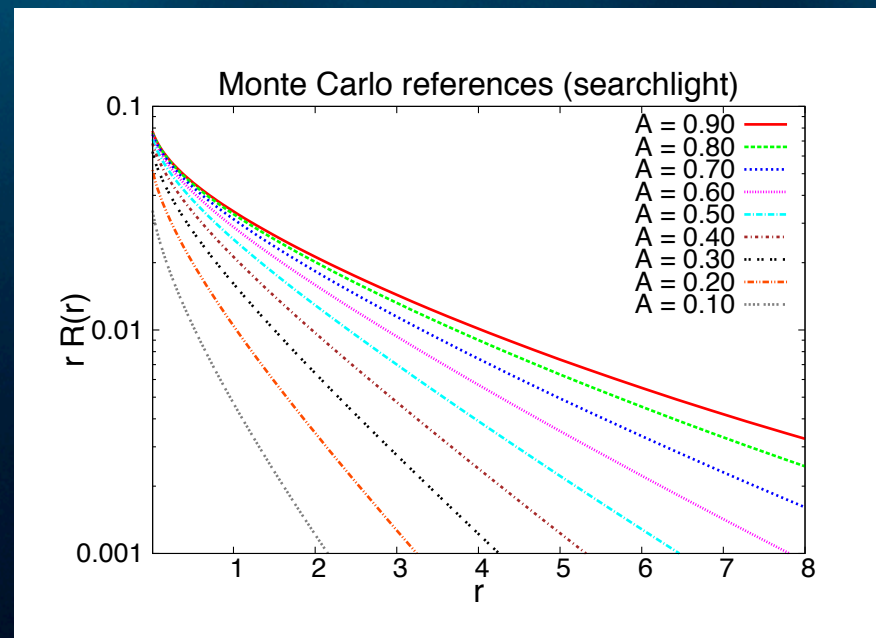
$$+ \frac{1}{N} \sum_{j=1}^N \frac{f(\vec{x}, \vec{\omega}, t_j) w_{\text{eq}}(t_j, \mathbf{x})}{w_{\text{exp}}(t_j, \mathbf{x}) \text{pdf}_{\text{eq}}(t_j | \vec{x}, \vec{\omega})}$$

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$$r^{(1)}(\vec{x}, \vec{x}_r(t)) = \frac{f_s(\vec{\omega} \cdot \vec{\omega}_{\vec{x}_r, \vec{x}}) e^{-\sigma_t(d_r(t))} F_t(\theta_o, \eta) F_t(\theta_i, 1/\eta) \cos \theta_o}{d_r^2(t)}$$

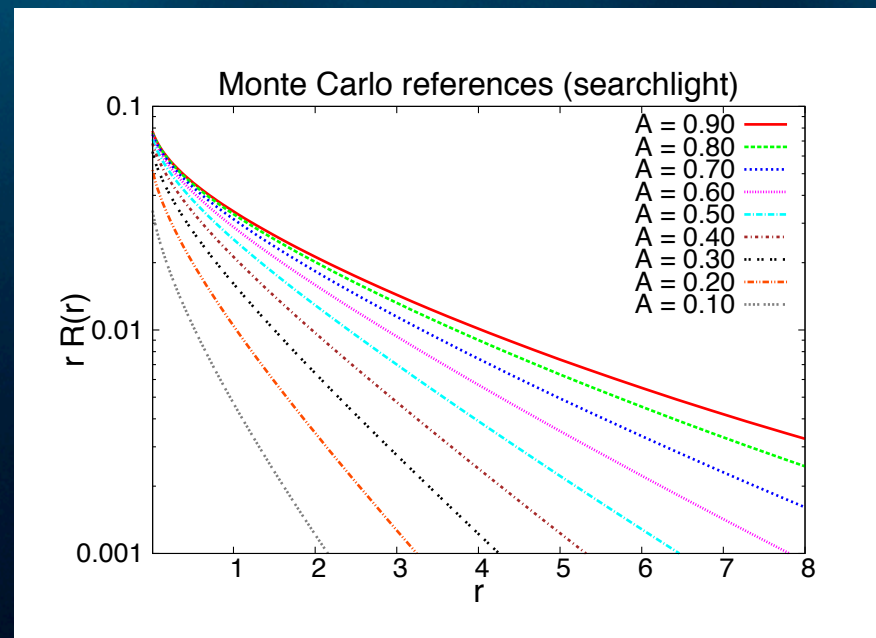
Approximate reflectance profiles

- Forget physics ... just approximate curves!
- Standard approach: sum of Gaussians



Approximate reflectance profiles

- Burley: curves look more like exponentials
- Sum of two exponentials (divided by distance r) is remarkably good approximation



Approximate reflectance profiles

- Normalized diffusion model [Burley]:

$$R(r) = \frac{e^{-r/d} + e^{-r/(3d)}}{8\pi d r}$$

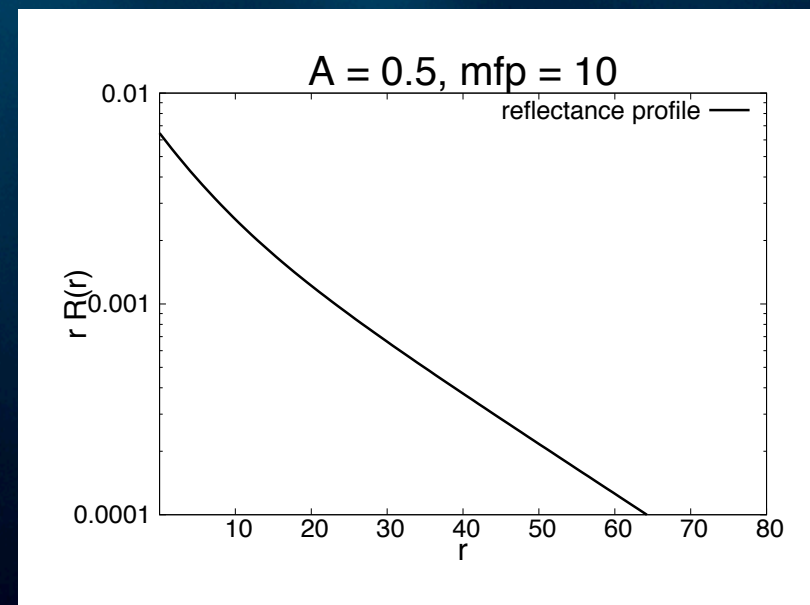
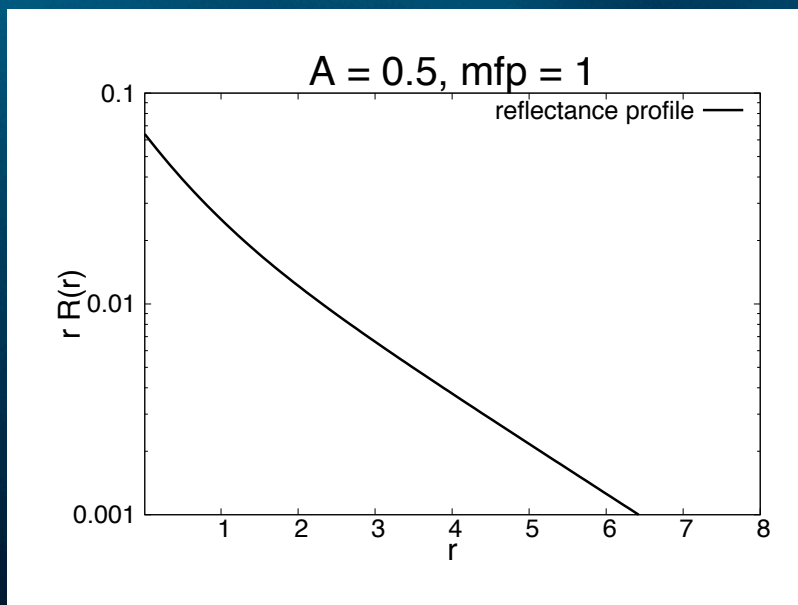
- Multiply by A = surface albedo
- d controls width and height of curve ... but what is d ??
 - artistic control of subsurface “softness”
 - what is connection between d and physical params?

Translation from physical param to d

- Our usual way of expressing scattering distance is mfp or dmfp:
 - mean free path in volume
 - diffuse mean free path on surface
- Let's find a "translation" s between mfp and d :
 - $d = \text{mfp} / s$ (s depends on A)
- With a translation we can compare normalized diffusion with physically-based diffusion models

Translation from physical param to d

- To determine s it is sufficient to consider only curves for $mfp=1$ since the shape of reflectance profile curve for given A is independent of mfp



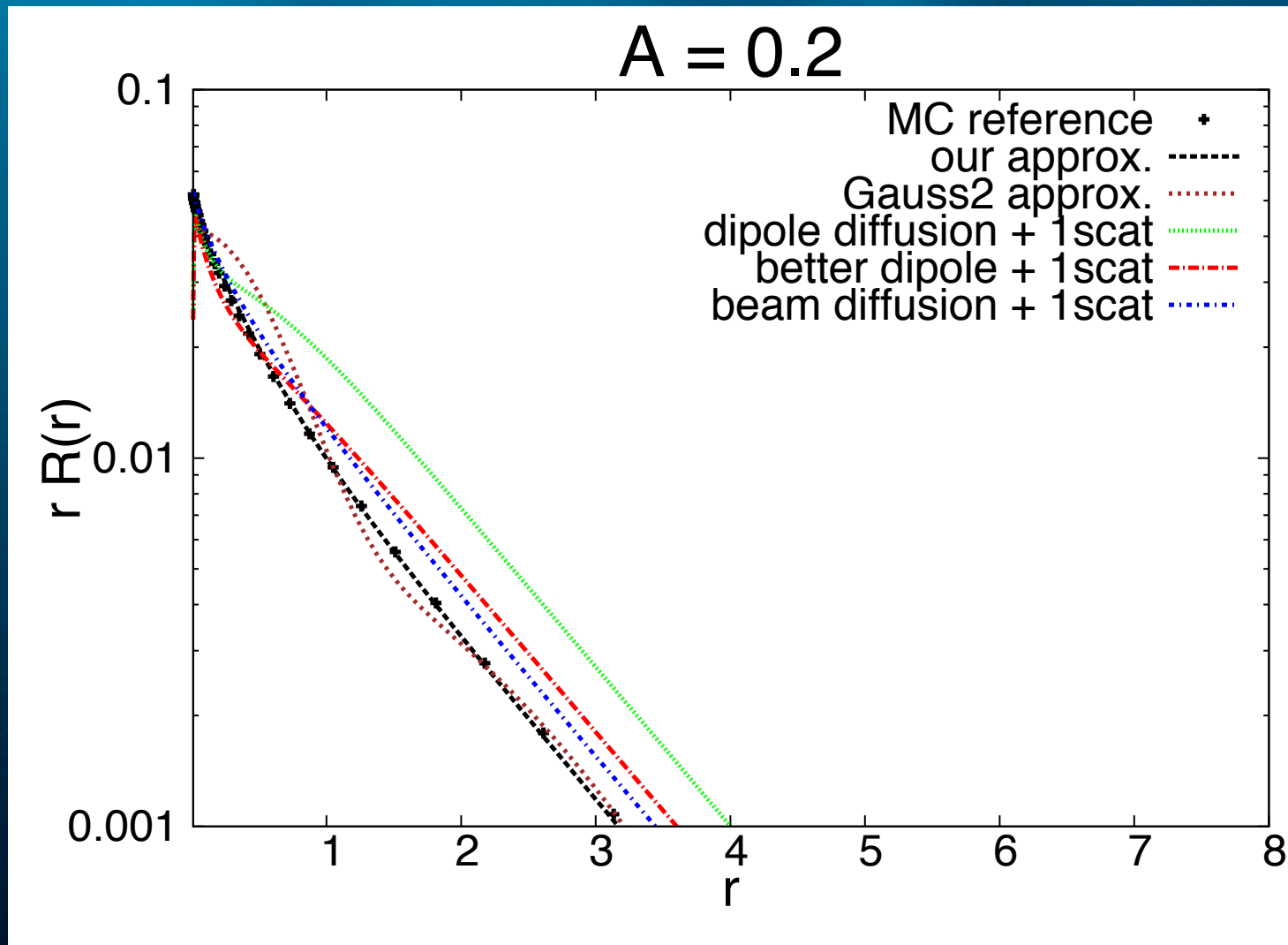
Translation from mfp to d

- For mfp=1 :

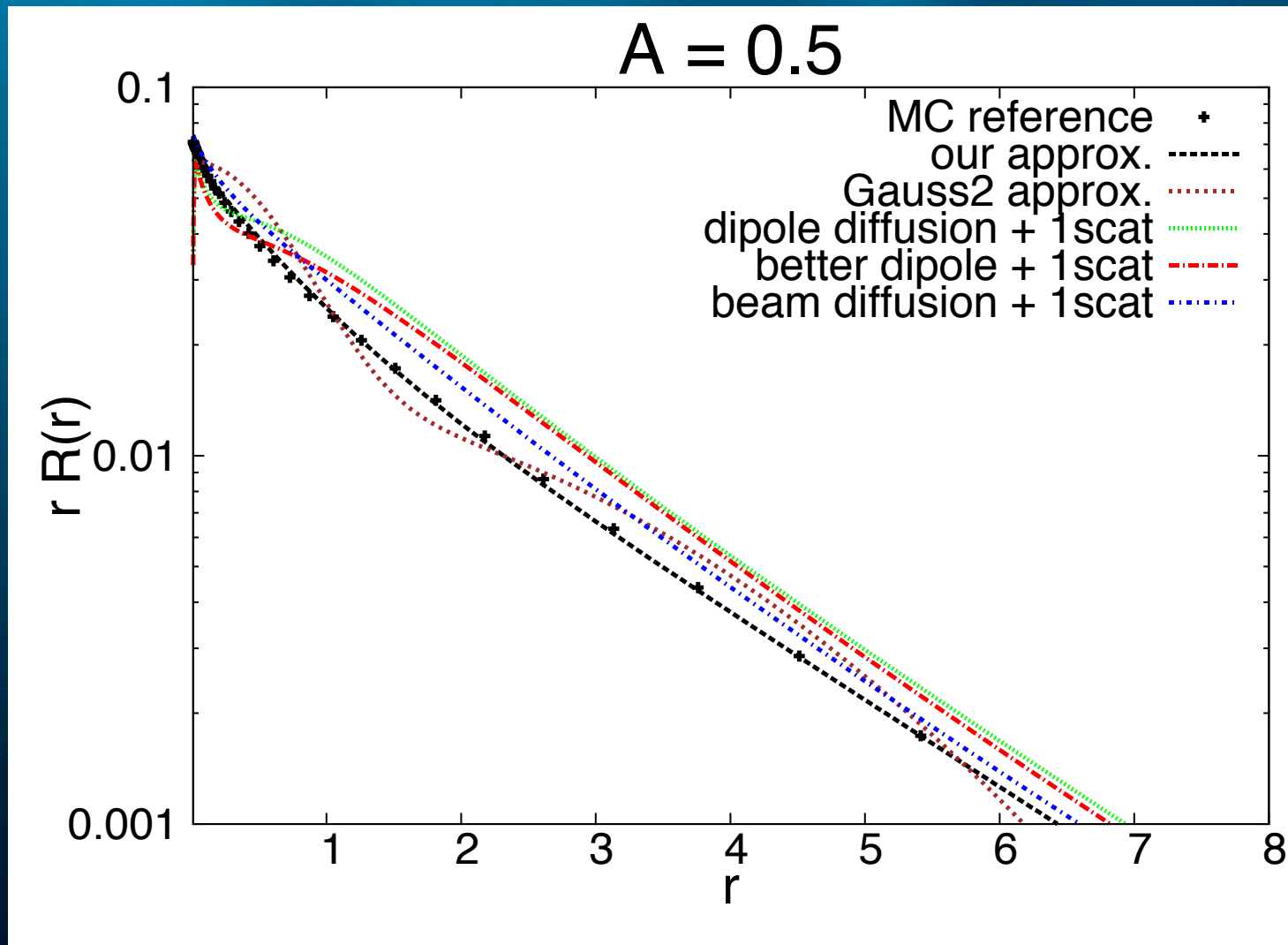
$$R_{\ell=1}(r) = A s \frac{e^{-sr} + e^{-sr/3}}{8\pi r}$$

- Find s that minimizes difference between $R(r)$ and Monte Carlo reference for same A
- For example: with optimal s for $A = 0.2, 0.5, 0.8 \dots$:

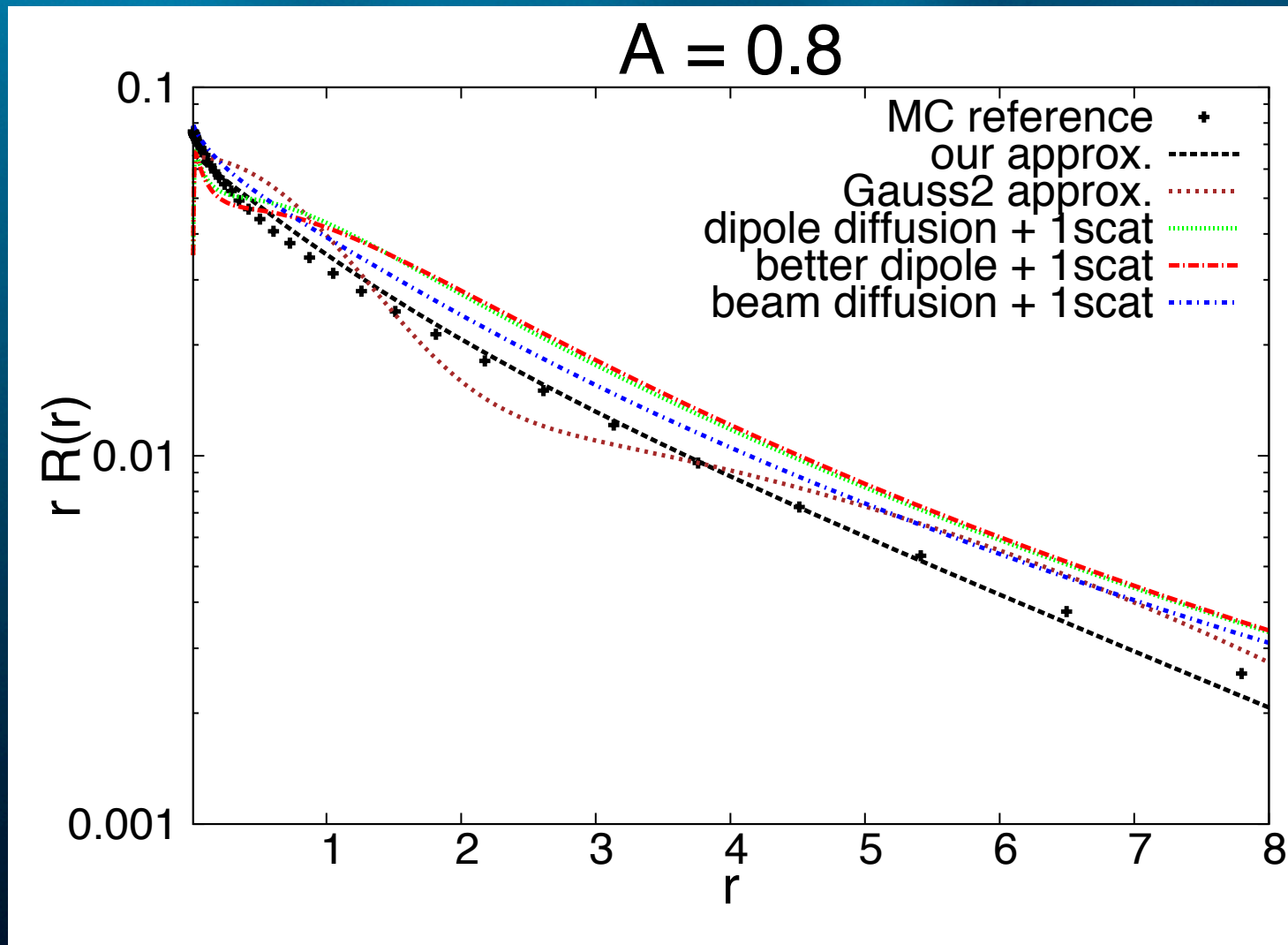
Comparisons: surface albedo 0.2



Comparisons: surface albedo 0.5



Comparisons: surface albedo 0.8



Comparisons: summary

- Normalized diffusion is closer to the MC reference points than dipole, better dipole, beam diffusion (w/ single scattering)
- Normalized diffusion (two exponentials) is a better approximation than two Gaussians

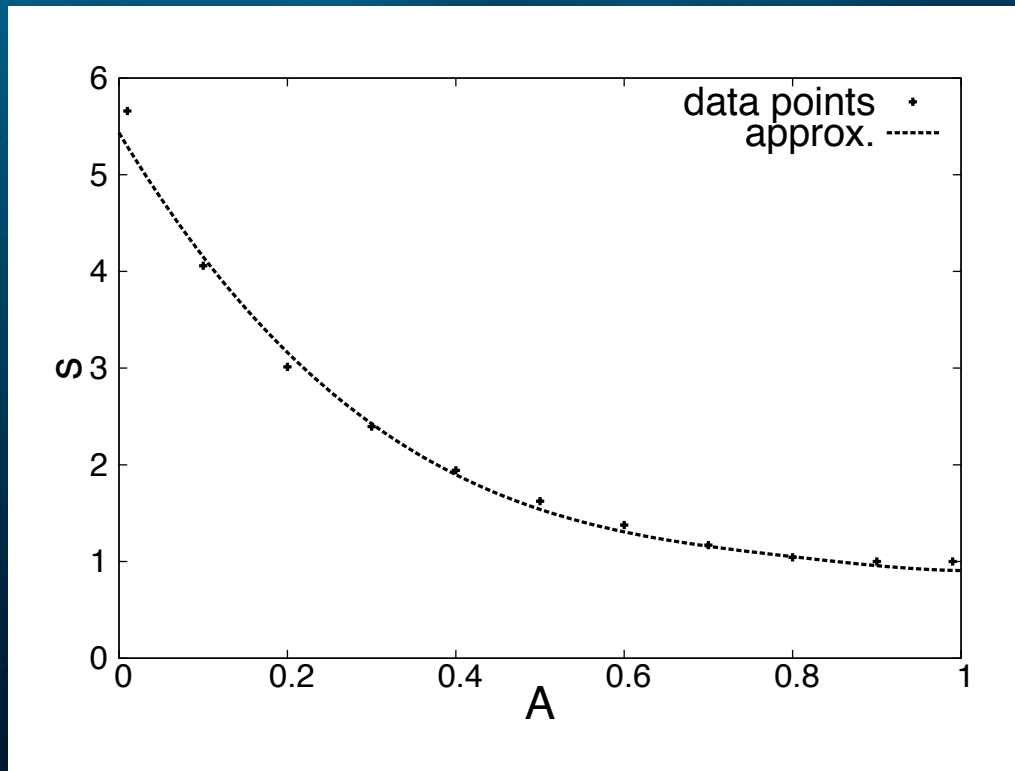
Translation from mfp to d

- Find s that minimizes difference between $R(r)$ and Monte Carlo reference for all A in 0.01, 0.02, ... , 0.99
- Gives data points; fit simple polynomial

Translation from mfp to d

- Data points and fitted curve:

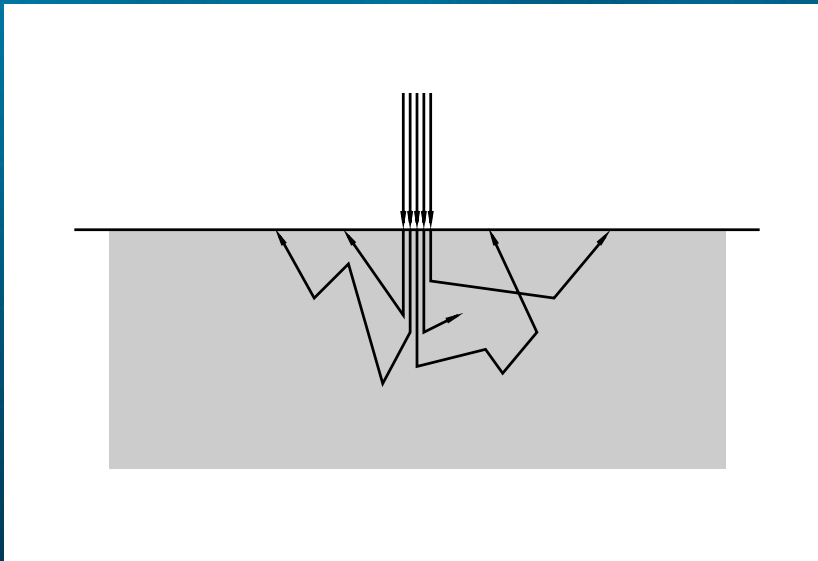
$$s = 1.85 - A + 7|A - 0.8|^3$$



Translation from mfp to d

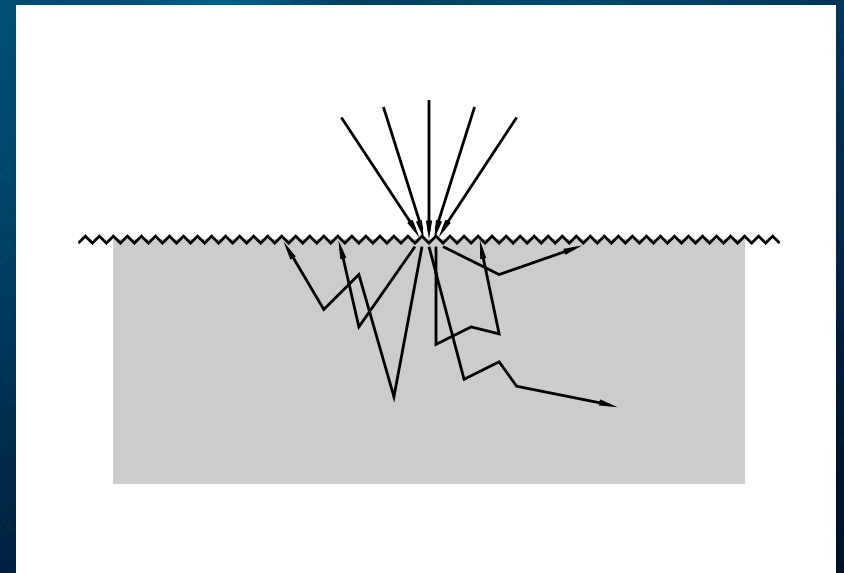
- Error wrt. MC references is $\sim 5.5\%$
- Small error compared to approximations and assumptions built into MC references: semi-infinite homogeneous volume, flat surface, ...

Diffuse surface transmission



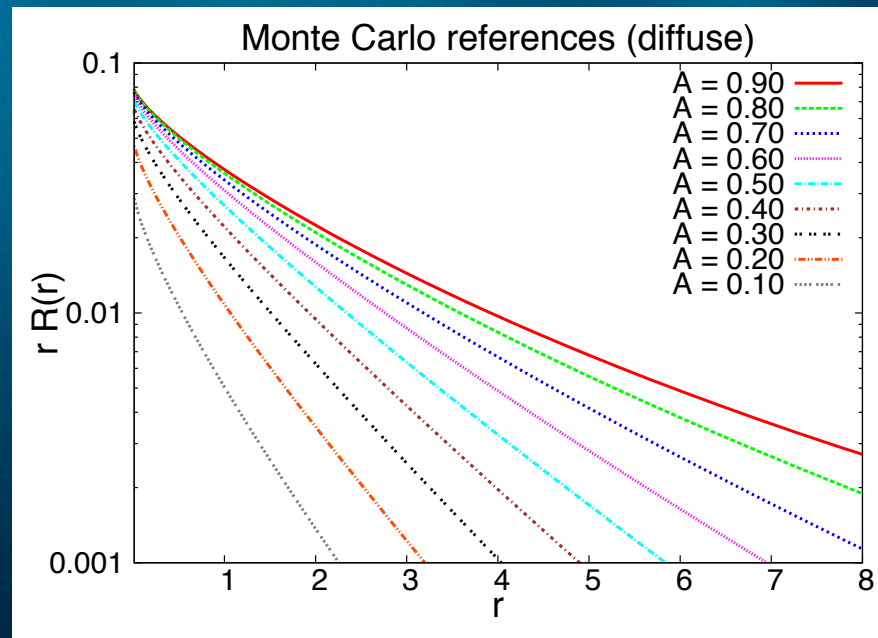
searchlight configuration
milk, juice, oily skin, ...

vs.



diffuse transmission
dry skin, make-up, ...

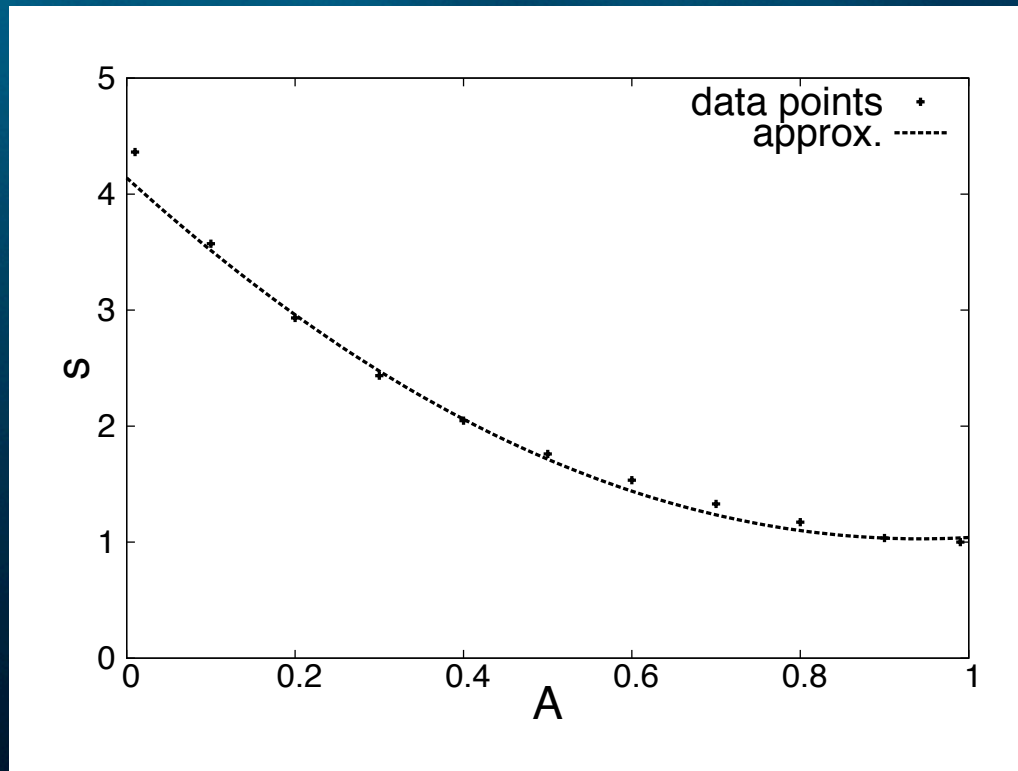
Diffuse surface transmission



Translation from mfp to d (diffuse)

- Data points and fitted curve:

$$s = 1.9 - A + 3.5(A - 0.8)^2$$



Translation from mfp to d (diffuse)

- Error wrt. MC references is only $\sim 3.9\%$
- In practical use: not much visual difference between searchlight approx and diffuse-transmission approx -- even though built on very different assumptions

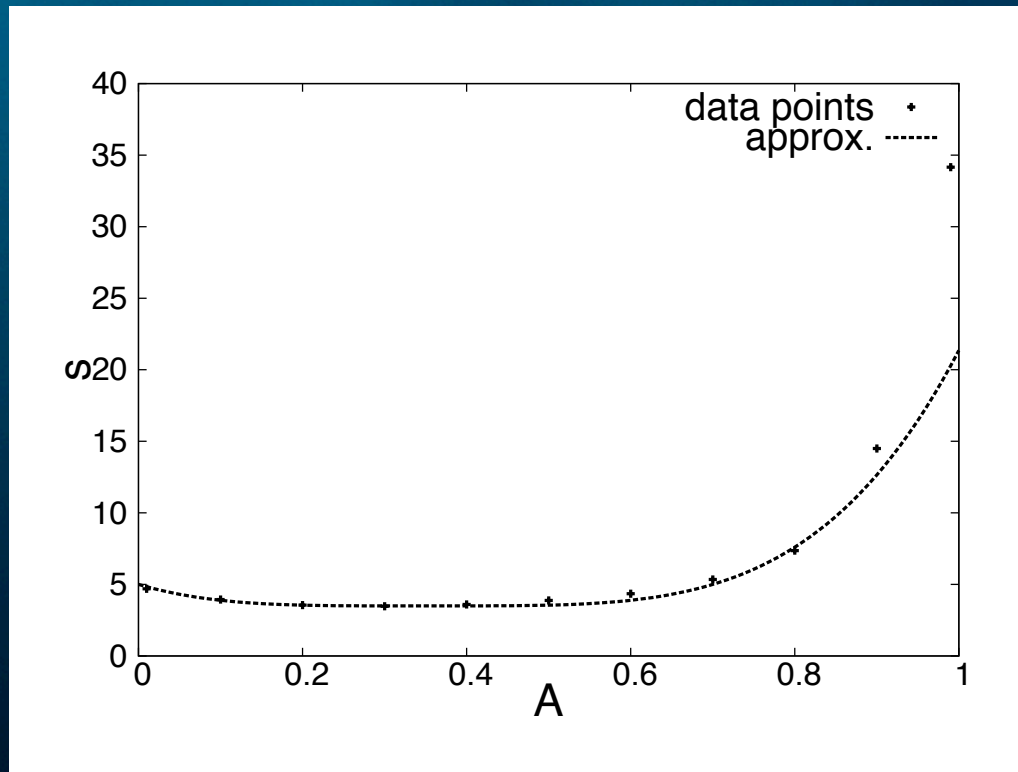
Translation from dmfp to d

- Back to searchlight configuration
- Change parameterization of scattering distance: diffuse mean free path on surface (instead of mean free path in volume)

Translation from dmfp to d

- Data points and fitted curve:

$$s = 3.5 + 100 (A - 0.33)^4$$



Translation from dmfp to d

- Error wrt. MC references is $\sim 7.7\%$
- In practical use: dmfp might be more intuitive than mfp; hence standard parameter of our previous scattering models

Translation summary

- 3 ways to determine d in Burley's normalized diffusion formula:
 - mfp to d for searchlight configuration
 - mfp to d for diffuse transmission
 - dmfp to d for searchlight configuration
- 3 simple polynomials for $s = s(A)$
- Pick the one you like!

Practical detail: importance sampling

- Importance sampling of distance r between light entry and exit points: need $\text{cdf}(r)$
- For physically-based BSSRDFs the cdf has to be computed with numerical integration: slow
- Burley's normalized diffusion has simple cdf:

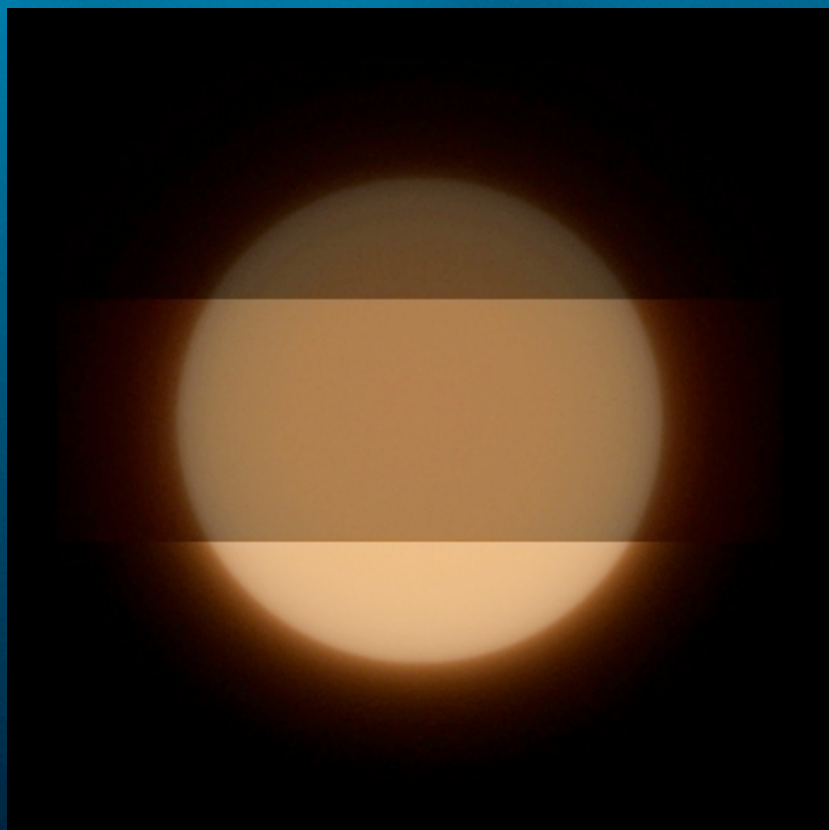
$$\text{cdf}(r) = 1 - \frac{1}{4}e^{-sr/\ell} - \frac{3}{4}e^{-sr/(3\ell)}$$

Discussion

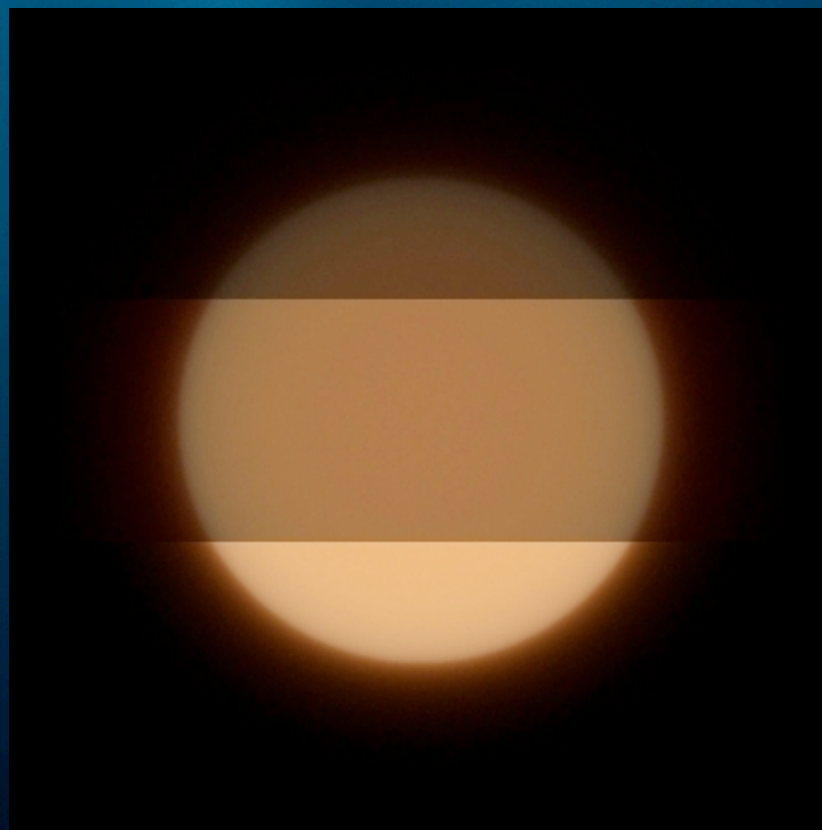
- Much simpler than physically-based diffusion (e.g. quantized diffusion or beam diffusion)
- Many times faster*

- *footnote: only a bit faster if careful table-based optimizations of physically-based

Result: comparison w/ beam diffusion



beam diffusion + 1scatter

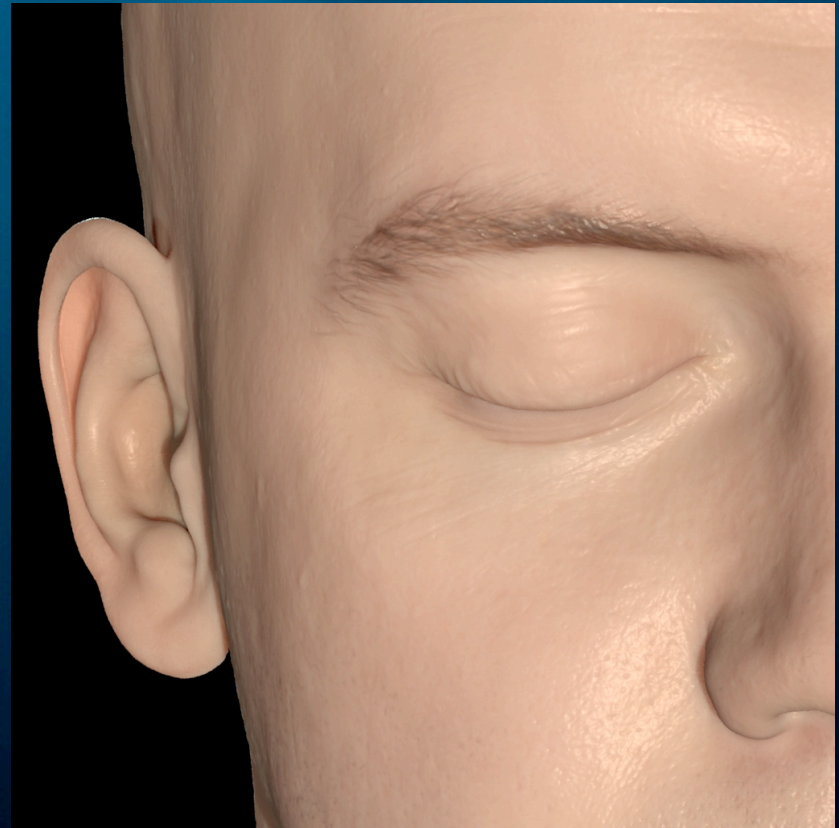


our approx

Result: comparison w/ beam diffusion



beam diffusion + 1scatter



our approx

(Head data: Infinite Realities)

PIXAR

Results



marble



fruits



plastic

Result: still life



image credit: Dylan Sisson

Conclusion

- Reparameterization of Burley's normalized diffusion approximation gives plug-in replacement of physically-based diffusion formulas -- same parameters
- Simpler, faster
- Error wrt. MC references is only a few percent
- More accurate than physically-based models
- One of the sss models built into RenderMan

Future work

- Oblique angles of incidence; non-symmetric scattering
 - maybe just s that depends on polar and relative azimuthal angle of incident illumination?
- Anisotropic scattering?

More information

- Burley, “Extending Disney’s physically based BRDF with integrated subsurface scattering”, Physically Based Shading Course
- Technical report: Christensen & Burley, graphics.pixar.com/library/ApproxBSSRDF

Acknowledgments

- HUGE thanks: Brent Burley
- Colleagues in RenderMan team
- Christophe Hery and Ryusuke Villemin (Pixar)
- Wojciech Jarosz and Ralf Habel (Disney)
- Images: Dylan Sisson, ...

Thank you!



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