Harmonic Coordinates for Character Articulation

Pixar Animation Studios
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Character Articulation
Direct Mesh Manipulation

Sorkine et al. 2004

Igarashi et al. 2005

Sumner et al. 2005
Volumetric Deformation

Character embedded in volume

Deform character by deforming volume

Decouple character geometry from articulation
Freeform Deformation

Barr 1984, Sederberg and Parry 1986

Cubical Grid: Topological Restrictions!
Freeform Deformation

Cubical Grid: Topological Restrictions!
Mean Value Coordinates

Ju, Schaeffer, Warren. SIGGRAPH 2005

Topologically Flexible Deformation System
Barycentric Coordinates

Piecewise linear on boundary, Smooth, Sum to 1

\[ p = \sum_{i=1,2,3} \beta_i(p) V_i \]

\[ p' = \sum_{i=1,2,3} \beta_i(p) V'_i \]
Generalized Barycentric Coordinates

Piecewise linear on boundary, Smooth, Sum to 1

\[ p = \sum_{i=1,\ldots,n} \beta_i(p)V_i \]

\[ p' = \sum_{i=1,\ldots,n} \beta_i(p)V_i' \]
Mean Value Coordinates

Floater 2003, Ju et al. 2005

Piecewise linear on boundary

Straight line distance from boundary for interpolation

Closed form formula
Mean Value Coordinates
large concavity produces *non-local motion in opposite direction*
Mean Value Coordinate Field

- Positive
- Negative

Significant negative weight
Desired Coordinate Field

- Positive
- Undefined
Laplace Equation for Interpolation

Steady-state heat equation

For every cage vertex $V_i$

solve Laplace Equation

$$\Delta h_i(P) = 0$$

$h_i(P)$ is *harmonic coordinate* of vertex $V_i$ at point $P$
Harmonic Coordinate Field

Weights drop-off with distance *within* cage

- **Positive**
- **Undefined**
Harmonic Coordinates

intuitive motion due to *interior locality and non-negativity*
Harmonic Coordinates

- Linear precision
- Sum to 1
- Reduce to barycentric coordinates for simplices
- Non-negative
- Interior locality
- Extended to $nD$
Numerical Solution

• No closed form: need numerical solution

  OK for character articulation!

• Finite Difference solution

• Regular grid

• Irregular Laplacian stencil near boundary
Linear System Solver

- Sparse linear system solve
- Many different solution techniques
  - Multigrid Solver (used for this talk)
  - Direct Solver (SuperLU)
Articulation of Production Character
Articulation of Production Character
Extensions for Additional Control

- Interior Control
- Dynamic Binding
Interior Control – Need for Blockers
Interior Control – With Blockers
Interior Control – Need for Subcage
Interior Control – Need for Subcage
Interior Control – With Subcage
Interior Control – Final
Dynamic Binding

Initial Pose  Bind  Final Pose
Pose Object within Cage  Pose Object by moving Cage
Dynamic Binding – Memory Costs

<table>
<thead>
<tr>
<th>Method</th>
<th>Memory Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Naive</td>
<td>100MB</td>
</tr>
<tr>
<td>Sparse</td>
<td>3MB</td>
</tr>
<tr>
<td>Statistics</td>
<td>Value 1</td>
</tr>
<tr>
<td>---------------------</td>
<td>---------</td>
</tr>
<tr>
<td># of cage vertices</td>
<td>112</td>
</tr>
<tr>
<td># of object vertices</td>
<td>8019</td>
</tr>
<tr>
<td>Grid Resolution</td>
<td>$32^3$</td>
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<tr>
<td>Solve Time (sec.)</td>
<td>17.6</td>
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<tr>
<td>(a preprocess)</td>
<td></td>
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<tr>
<td>Pose Time (sec.)</td>
<td>0.026</td>
</tr>
<tr>
<td>Grid Size (MB)</td>
<td>3.7</td>
</tr>
</tbody>
</table>
Summary

- Harmonic Coordinates – a new form of *generalized barycentric coordinates*

- Especially suitable for character articulation
  - Interior Locality
  - Non-negative

- Extensions for additional control in character animation pipeline
Harmonic Coordinates – Drawbacks

• No closed form formulation
  – *Interior locality and non-negativity are more important for character articulation.*

• Coordinates undefined on cage exterior

• Cage must be a bounded volume
Future Work

- Adaptive grids
- Moving cages
- Incremental solves
- “Positive Mean Value Coordinates” (Lipman et al. SGP 2007)
Thank you!