Reduced Boundary Viscosity for Smoke

Alexis Angelidis
Pixar Technical Memo #19-02

Abstract

We propose a method to reduce viscosity at the boundaries of incompressible flows. Our method allows artists to control the amount of boundary viscosity by mixing a new slip boundary solution with an existing no-slip solution. The cost of our method is an additional pre-computation and a post-computation that can run in parallel for every frame. The running cost of the main solver is not changed, and arguably faster, since reducing boundary viscosity tends to reduce the domain occupied by the fluid. Our method can produce both a velocity field for time integration, or a geometric deformation to bypass time integration.

1 Introduction

In Computer Graphics, boundaries often accelerate and self-intersect in physically impossible ways, for justifiable reasons. Therefore, Visual Effects Artists must work very carefully with the velocity induced by boundary accelerations, and are experts at creating plausible interpretation of the fluid movement around those surfaces. For smoke simulation, the method of Stam [Stam 1999] is the method of choice, and it particularly good at handling boundaries. In this family of solvers, the numerical viscosity induced by the voxel size thickens the boundary layer. This makes large objects feel smaller. Artists spend also a significant amount of time on these areas, to reduce the perceived thickness. The aim of our paper is to provide artists with a general method for thinning the movement of fluids around surfaces.

It is perhaps surprising that an ideal fluid that is inviscid and incompressible does not accumulate the acceleration of boundaries over time, a physical behavior known to be displayed only by cool liquid helium. Even though ideal fluid is not representative of real fluids, it displays a familiar behavior. We use this property to run computations in parallel for every frame.

Our method is related to [DeRose and Mark 2006; Ben-Chen et al. 2006], except that we bypass entirely the time integral, thanks to the atemporal property of ideal fluids. Our method is also related to [Bridson et al. 2007; DeWolf 2006]: we model the deforming boundaries hinted in their future work.

2 Timeles Slip Boundary

In the absence of viscosity and external forces, the movement of an incompressible fluid simplifies greatly:

\[
\begin{align*}
\frac{\partial \mathbf{h}}{\partial t} &= - (\mathbf{h} \cdot \nabla) \mathbf{h} - \nabla p \\
\nabla \cdot \mathbf{h} &= 0 \\
\mathbf{n} \cdot \mathbf{h} &= 0 \text{ if } \mathbf{p} \in S \\
\mathbf{h} &= 0 \text{ if } \mathbf{p} \to \infty
\end{align*}
\]

If the initial velocity has no curl to begin with, then the fluid becomes atemporal. It is the solution to the following equations defined by the boundaries \(S\):

\[
\begin{align*}
\nabla \times \mathbf{h} &= 0 \\
\nabla \cdot \mathbf{h} &= 0 \\
\mathbf{n} \cdot \mathbf{h} &= 0 \text{ if } \mathbf{p} \in S \\
\mathbf{h} &= 0 \text{ if } \mathbf{p} \to \infty
\end{align*}
\]

We solve \(\mathbf{h}\) with two mappings that are inverses of each other: a deflating mapping \(f_h^{-1}\) at time \(t - \Delta t\), and an inflating mapping \(f_h\) at time \(t\). The deflating mapping \(f_h^{-1}\) is defined as the volume preserving deformation that follows the volume decreasing shape \(S(\mathbf{p}, t)\):

\[
\frac{\partial \mathbf{p}}{\partial t} = \nabla \iint_{S(\mathbf{p}, t)} \frac{1}{||\mathbf{p} - x||} \, dx
\]

The trajectory of \(\mathbf{p}\) ends naturally, since the volume of \(S(\mathbf{p}, t)\) must eventually vanish, although this may be in a topologically varying manner. Because the integrand is harmonic, both mappings are harmonic. And because both mappings are harmonic, the result is also harmonic by construction. Recovering the harmonic field \(\mathbf{h}\) from this mapping is straightforward, as shown in Figure 3:

\[
\mathbf{h}(\mathbf{p}) = \frac{f_h(f_h^{-1}(\mathbf{p}, t), t + \Delta t) - \mathbf{p}}{\Delta t}
\]

This vector field \(\mathbf{h}\) does not leak into the surface because the integrand is singular, and tears space apart. Also, we avoid the differential singularity of the harmonic field at the boundary by replacing the velocity by a discrete displacement. To discretize the mapping \(f_h^{-1}\), we create a series of meshes \(T_i\) per Algorithm 1 shown in Figure 2.

Figure 1: Left: unaltered simulation. Left to right: thinning of the boundary layer viscosity by mixing an atemporal ideal fluid.

Figure 2: Insertion of a boundary with 4 steps. The volume of the surrounding space is preserved and no rotation is introduced.
We propose in this paper a field can be found in the triangles. By storing the field on a grid, we avoid the singularity gradient or the triangle source field is singular along the edges of the triangles. To account and existing velocity field \( \mathbf{v} \), the harmonic fields must reflect that velocity along the normal \( \mathbf{n} \) of the boundary:

\[ \mathbf{v} \cdot \mathbf{n} = -\mathbf{h} \cdot \mathbf{n} \]

We propose a simple way to enforce this contrain by substituting the boundary \( M \) with \( M' \) in Algorithm 1, to make sure that the field does not leak:

\[ M' = \{ \mathbf{f}_h^{-1}(\mathbf{p}), \mathbf{p} \in M \} \]

With this replacement, any field will satisfy the boundary condition. The harmonic term can remain atemporal as long as the velocity term is atemporal. This also requires an irreversible velocity field, or at least the following approximation:

\[ \mathbf{f}_v^{-1} = \mathbf{p} - \Delta \mathbf{v} \]

Thus we can synthesize a no-through boundary condition for any solenoidal vector field. Note that adding this boundary condition can add a time dependency, since \( \mathbf{v} \) is usually time dependent. In this case, the boundary computation is only worth doing if flow dominates the boundary.

### 3 Reduced Boundary Viscosity

To reduce the numerical viscosity of an existing flow, we propose to deflate the boundary by an amount proportional the amount of viscosity that the artist wants to remove. Since the partial deflation and partial inflation can run in parallel for every frame as a pre-process and as a post-process, the main smoke solver is left unaltered.

### 4 Conclusion

Our method reduces the numerical viscosity of grid-based smoke solver boundaries. It can also be used with particle solvers, or to manipulate the incompressible lensing effect that occurs around surfaces that are embedded in a fluid. Since harmonic field is atemporal, our tool can be manipulated by the artist with minimal impact on lengthy temporal integrations.
References


