# Importance Resampling for BSSRDF

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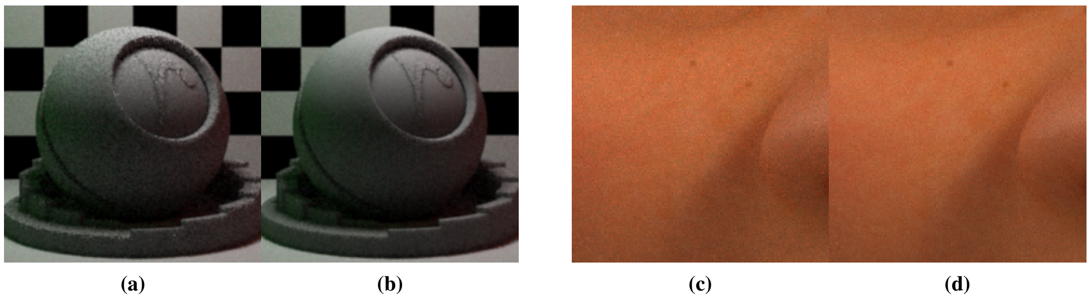


**Figure 1:** *BSSRDF without resampling (a)(c), with resampling (b)(d).*

Rendering translucent effects using the diffusion model BSSRDF [Jensen et al. 2001] is still a difficult problem compared to traditional BSDF, because it introduces additional degrees of freedom. In a BSSRDF, the light does not scatter at a single point: it enters the surface at one point, and exits at another. Various techniques have been proposed to sample an exit point based on a entry point. Traditionally, we prebuilt a point cloud [Jensen and Buhler 2002], distributed on the surface of the object, and when rendering we would use it as exit points. However this had multiple problems, for small DMFPs, the density of the point cloud could be arbitrary high, and the heavy precomputation was not pathtracing friendly.

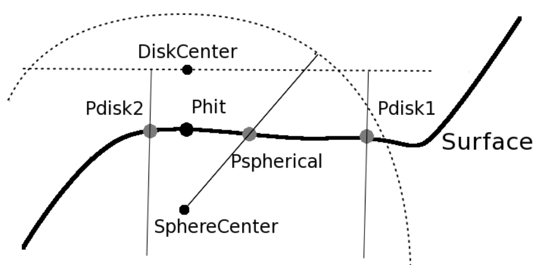


**Figure 2:** *Before resampling, 1 spherical sample, 2 disk samples.*

Nowadays, on-the-fly sampling based on raytracing (with recent improvements [King et al. 2013]) are preferred. Unfortunately, these methods add additional variance as it is not easy to perfectly importance sample an exit point based on the diffusion model. For example, it is not possible to analytically integrate and invert the original dipole, and even with more recent models [Christensen 2015], it is difficult to sample the distance taking into account the geometry of the surface at the same time. Because of that, the CDF inversion is not perfect and leads to additional noise when compared to a simple diffuse illumination.

We propose to use **importance resampling** [Talbot et al. 2005] to improve the distance sampling part of those models. This method is independent of the sampling method (spherical, disk, etc), and also independent of the model itself (dipole, better dipole, etc). It can be used in conjunction with any of the aboves (MIS or not), is trivial to implement in any raytracer, and compatible with pathtracing as it does not require expensive path splits.

Importance resampling is a technique used when we need to generate a sampling distribution $g$, where no analytical solution is found because there is no way to integrate and invert $g$. When using resampling, we first generate a set of samples $X$ from a calculable pdf $p$. Then we weight each sample $x_i$ by the weight $w_i = g(x_i)/p(x_i)$, then pick randomly a subset of samples $Y$ from $X$ using those weights. By doing so, we ensure that the final set $Y$ is distributed according to $g$ (exactly if $X$ is infinite, approximate if not).

For the **diffusion case**, reusing the original paper's notation, we set $g$ to be the result of the diffusion function, $p$ will be the PDF used to generate those samples. Finally, we get a set of samples $Y$ that is approximately distributed according to the diffusion function.At Pixar, we resample using a set of samples $X$ that was generated using MIS between disk and spherical techniques, but also between RGB channels for highly saturated DMFPs.

In practice there is a cost associated with the generation of the sample set $X$. This cost is amortized because the generation only requires probe rays that won't trigger any additional shading. And in production shots with complex shaders, it is not rare that the main rendering cost is shading. In addition, when tracing those probe rays to find candidate exit points, we use a trace subset which is usually restricted to the surface we are shading, and the maximum tracing distance is also kept tight according to the DMFP.

## References

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