Reduced Boundary Viscosity for Smoke

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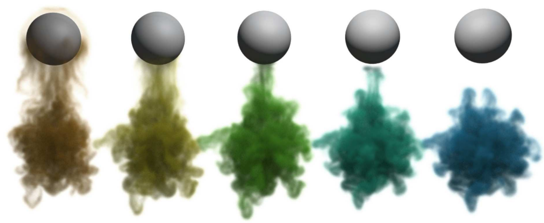

Figure 1: *Left: unaltered simulation. Left to right: thinning of the boundary layer viscosity by mixing an atemporal ideal fluid.*

Abstract

We propose a method to reduce viscosity at the boundaries of incompressible flows. Our method allows artists to control the amount of boundary viscosity by mixing a new slip boundary solution with an existing no-slip solution. The cost of our method is an additional pre-computation and a post-computation that can run in parallel for every frame. The running cost of the main solver is not changed, and arguably faster, since reducing boundary viscosity tends to reduce the domain occupied by the fluid. Our method can produce both a velocity field for time integration, or a geometric deformation to bypass time integration.

1 Introduction

In Computer Graphics, boundaries often accelerate and self-intersect in physically impossible ways, for justifiable reasons. Therefore, Visual Effects Artists must work very carefully with the velocity induced by boundary accelerations, and are experts at creating plausible interpretation of the fluid movement around those surfaces. For smoke simulation, the method of Stam [Stam 1999] is the method of choice, and it particularly good at handling boundaries. In this family of solvers, the numerical viscosity induced by the voxel size thickens the boundary layer. This makes large objects feel smaller. Artists spend also a significant amount of time on these areas, to reduce the perceived thickness. The aim of our paper is to provide artists with a general method for thinning the movement of fluids around surfaces.

It is perhaps surprising that an ideal fluid that is inviscid and incompressible does not accumulate the acceleration of boundaries over time, a physical behavior known to be displayed only by cool liquid helium. Even though ideal fluid is not representative of real fluids, it displays a familiar behavior. We use this property to run computations in parallel for every frame.

Our method is related to [DeRose and Mark 2006; Ben-Chen et al. 2006], except that we bypass entirely the time integral, thanks to the atemporal property of ideal fluids. Our method is also related to [Bridson et al. 2007; DeWolf 2006]: we model the deforming boundaries hinted in their future work.

2 Timeles Slip Boundary

In the absence of viscosity and external forces, the movement of an incompressible fluid simplifies greatly:

$$\begin{cases} \frac{\partial \mathbf{h}}{\partial t} &= -(\mathbf{h}\cdot\nabla)\mathbf{h} - \nabla p \\ \nabla\cdot\mathbf{h} &= 0 \end{cases} \tag{1}$$

If the initial velocity has no curl to begin with, then the fluid becomes atemporal. It is the solution to the following equations defined by the boundaries S:

$$\begin{cases} \nabla\times\mathbf{h} &= 0 \\ \nabla\cdot\mathbf{h} &= 0 \\ \mathbf{n}\cdot\mathbf{h} &= 0 \text{ if } \mathbf{p}\in \mathrm{S} \\ \mathbf{h} &= 0 \text{ if } \mathbf{p}\to\infty \end{cases} \tag{2}$$

We solve $\mathbf{h}$ with two mappings that are inverses of each other: a deflating mapping $\mathbf{f}_h^{-1}$ at time $t-\Delta_t$, and an inflating mapping $\mathbf{f}_h$ at time t. The deflating mapping $\mathbf{f}_h^{-1}$ is defined as the volume preserving deformation that follows the volume decreasing shape $S(\mathbf{p},t)$:

$$\frac{\partial \mathbf{p}}{\partial t} = \nabla \iiint_{\mathrm{S}(\mathbf{p},t)} \frac{1}{\|\mathbf{p}-\mathbf{x}\|}\mathrm{d}\mathbf{x} \tag{3}$$

The trajectory of $\mathbf{p}$ ends naturally, since the volume of $\mathrm{S}(\mathbf{p},t)$ must eventually vanish, although this may be in a topologically varying manner. Because the integrand is harmonic, both mappings are harmonic. And because both mappings are harmonic, the result is also harmonic by construction. Recovering the harmonic field $\mathbf{h}$ from this mapping is straightforwad, as shown in Figure 3:

$$\mathbf{h}(\mathbf{p}) = \frac{\mathbf{f}_h(\mathbf{f}_h^{-1}(\mathbf{p},t), t+\Delta_t) - \mathbf{p}}{\Delta_t} \tag{4}$$

This vector field $\mathbf{h}$ does not leak into the surface because the integrand is singular, and tears space apart. Also, we avoid the differential singularity of the harmonic field at the boundary by replacing the velocity by a discrete displacement. To discretize the mapping $\mathbf{f}_h^{-1}$, we create a series of meshes T_i per Algorithm 1 shown in Figure 2.

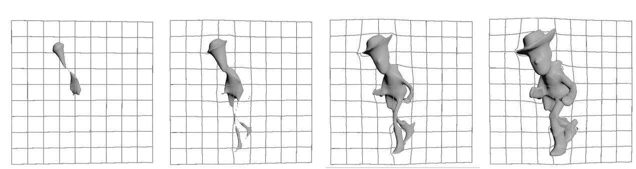

Figure 2: *Insertion of a boundary with 4 steps. The volume of the surrounding space is preserved and no rotation is introduced.*

Algorithm 1 Sources(d,n,s,M)

Require: d = depth input parameter
Require: n = steps input parameter
Require: s = voxel size input parameter
Require: M = mesh input geometry
for $i \in [1, n]$ **do**
  S_i = voxels with signed distance of M
  C = explicit triangulation of 0-level set of S_i
  V_C = volume of C
  T_i = erode C explicitly using $-\frac{d}{n}\nabla \mathrm{S}_i(\mathbf{p})$
  V_T = volume of T_i
  A_T = area of T_i
  $k_i = \frac{V_C - V_T}{4\pi A_T}$
  M = shrink M by source field induced by T_i amount k_i
end for
return $\{\mathrm{T}_i, i \in [1, n]\}$, $\{k_i, i \in [1, n]\}$

The amount of shrinkage k_i in Algorithm 1 is an upper bound to prevent the outer space of T_i from crossing the boundary, and it is found by inserting the volume of a sphere $\frac{4\pi}{3}d^3$ in the analytical solution of a point weighted by the area of the source [Decaudin 1996] where $3k_i = d^3$:

$$\mathbf{f}_h(\mathbf{p}, k_i) = (1 + \frac{3k_i}{\|\mathbf{p}\|^3})^{1/3}\mathbf{p} \tag{5}$$

$$\frac{\mathrm{d}}{\mathrm{d}k_i}\mathbf{f}_h(\mathbf{p}, k_i)\bigg|_{k_i=0} = \frac{1}{\|\mathbf{p} - \mathbf{x}\|} \tag{6}$$

This bound works because the speed induced by a point sink is faster the closer we are to the sink. We can then use the meshes T_i and amounts k_i to compute the deflating mapping:

$$\mathbf{p}_{i+1} = k_i \nabla \oiint_{\mathrm{T}_i} \frac{1}{\|\mathbf{p}_i - \mathbf{x}\|}\mathrm{d}\mathbf{x} \tag{7}$$

The inflating mapping is simply achieved by reversing the order of the meshes, and using $-k_i$ instead of k_i. The integral of Equation 7 can be computed in a variety of manners, and the FMM (fast multipole method) is strongly advised for pratical situations involving multiple characters. The FMM coefficients for integrating a source field can be found in [Angelidis 2017]. We propose in this paper a non-scalable and stable method in Algorithm 2, for the reader looking for a simpler starting point:

Algorithm 2 Inflate($\mathbf{p}$, T_i, k_i) procedure for $f_h(\mathbf{p})$

Require: $\mathbf{p}$
Require: T_i
Require: k_i
1: if $\mathbf{p}$ is extremely close to T_i, then compute $k_i \oiint_{\mathrm{T}_i} \frac{1}{\|\mathbf{p}-\mathbf{x}\|}\mathrm{d}\mathbf{x}$ in a voxel narrow band of T_i and compute a discrete gradient from the voxels.
2: if $\mathbf{p}$ is close enough to T_i, then compute $k_i \oiint_{\mathrm{T}_i} \nabla \frac{1}{\|\mathbf{p}-\mathbf{x}\|}\mathrm{d}\mathbf{x}$ analytically.
3: if $\mathbf{p}$ is far from T_i, then compute the integral as $k_i a_i \nabla \frac{1}{\|\mathbf{p}-\mathbf{c}_i\|}$, where $\mathbf{c}_i$ and a_i are the centroid and the area of T_i.
4: **return** a

The reason for step 1. in Algorithm 2 is because the analytical gradient or the triangle source field is singular along the edges of the triangles. By storing the field on a grid, we avoid the singularity in the regions that happen to be where the shrinking boundary is about to vanish.

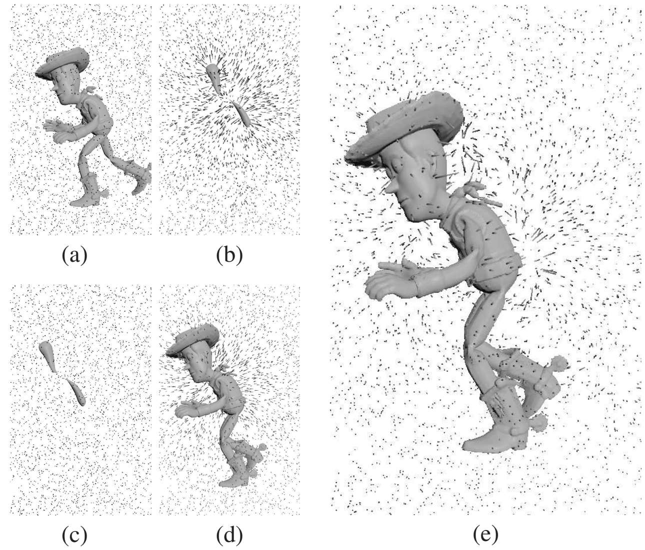

Figure 3: *(a)→(b): The boundary at $t - \Delta_t$ is deflated using $\mathbf{f}_h^{-1}$. (c)→(d): The boundary at t is inflated using $\mathbf{f}_h$. (e) The vector field from $t - \Delta_t$ to t is defined by the steps (a)→(b)→(c)→(d)*

Boundary Condition To account and existing velocity field $\mathbf{v}$, the harmonic fields must reflect that velocity along the normal $\mathbf{n}$ of the boundary:

$$\mathbf{v} \cdot \mathbf{n} = -\mathbf{h} \cdot \mathbf{n} \tag{8}$$

We propose a simple way to enforce this contrain by substituting the boundary M with M′ in Algorithm 1, to make sure that the field does not leak:

$$\mathrm{M}' = \{\mathbf{f}_v^{-1}(\mathbf{p}), \mathbf{p} \in \mathrm{M}\} \tag{9}$$

With this replacement, any field will satisfy the boundary condition. The harmonic term can remain atemporal as long as the velocity term is atemporal. This also requires an inversible velocity field, or at least the following approximation:

$$\mathbf{f}_v^{-1} = \mathbf{p} - \Delta_t \mathbf{v} \tag{10}$$

Thus we can synthesize a no-through boundary condition for any solenoidal vector field. Note that adding this boundary condition can add a time dependency, since $\mathbf{v}$ is usually time dependent. In this case, the boundary computation is only worth doing the if flow dominates the boundary.

3 Reduced Boundary Viscosity

To reduce the numerical viscosity of an existing flow, we propose to deflate the boundary by an amount proportional the amount of viscosity that the artist wants to remove. Since the partial deflation and partial inflation can run in parallel for every frame as a pre-process and as a post-process, the main smoke solver is left unaltered.

4 Conclusion

Our method reduces the numerical viscosity of grid-based smoke solver boundaries. It can also be used with particle solvers, or to manipulate the incompressible lensing effect that occurs around surfaces that are embeded in a fluid. Since harmonic field is atemporal, our tool can be manipulated by the artist with minimal impact on lengthy temporal integrations.

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