Texture mapping for the Better Dipole model

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1 Introduction

Eugene d'Eon followed up his seminal Quantized-Diffusion model [DI11] with a more practical Better Dipole [D'E12], which seems to preserve most of the qualities of the former translucency approach.

As pointed out in Implementing a Skin BSSRDF (chapter 4 of [Ren03]), the advantage of Jensen's original Dipole model [JMLH01] is that it could be reparametrized to allow easy texture mapping.

This short note is about re-deriving a similar inversion formula for the Better Dipole. This makes the new model a drop-in replacement for the old.

2 Integral R_d

Eugene d'Eon is proposing the following new diffusion profile (his Better Dipole):

$$R(r) = \frac{(\alpha')^2}{4\pi}\left[\left(C_{\hat{E}}\frac{z_r(\mu_{tr}d_r+1)}{d_r^2}+\frac{C_\phi}{D}\right)\frac{e^{-\mu_{tr}d_r}}{d_r}-\left(C_{\hat{E}}\frac{z_v(\mu_{tr}d_v+1)}{d_v^2}+\frac{C_\phi}{D}\right)\frac{e^{-\mu_{tr}d_v}}{d_v}\right] \quad (1)$$

We here compute the diffuse integral of R, which we call R_d, as:

$$R_d = 2\pi\int_0^\infty R(r)r\,dr \quad (2)$$

This resolves analytically exactly to:

$$R_d = \frac{(\alpha')^2}{2}e^{-\sqrt{\frac{3(1-\alpha')}{2-\alpha'}}}\left[C_{\hat{E}}\left(1+e^{-4A\mu_{tr}D}\right)+\frac{C_\phi}{\mu_{tr}D}\left(1-e^{-4A\mu_{tr}D}\right)\right] \quad (3)$$

with

$$\mu_{tr}D = \sqrt{\frac{(1-\alpha')(2-\alpha')}{3}} \quad (4)$$

R_d is what we numerically invert from the albedo to derive α'.

3 Numerical inversion of R_d

On the next page is our current implementation of this inversion technique.

Note that the coefficients C_1 and C_2 are functions of the index of refraction η and are given in [DI11, D'E12].

```
float computeRd(float alpha_prime, twoC1, threeC2)
{
    float Cphi  = 0.25 * ( 1.0 - twoC1 );
    float Ce    = 0.5  * ( 1.0 - threeC2 );
    float fourA = ( 1.0 + threeC2 ) / Cphi;
    float mu_tr_D = sqrt( ( 1.0 - alpha_prime ) * ( 2.0 - alpha_prime ) / 3.0 );
    float myexp = exp( -fourA * mu_tr_D );
    float Rd = 0.5 * alpha_prime * alpha_prime
                  * exp( - sqrt( 3.0 * ( 1.0 - alpha_prime ) / ( 2.0 - alpha_prime ) )
                  * ( Ce * ( 1.0 + myexp ) + Cphi / mu_tr_D * ( 1.0 - myexp ) );
    return Rd;
}

// Numerically invert the Rd formula
// to solve for reduced albedo alpha' given the albedo Rd and the internal
// parameters C1 and C2.
// The formula is highly non-linear but monotonically increasing.  Solved using bisectio
// (Secant root finding would be more efficient. Or use a precomputed table.)
float computeAlphaPrime(float Rd, twoC1, threeC2)
{
   int i, niter = 50; // number of iterations in bisection
   float x0 = 0, x1 = 1;
   float xmid, fmid;

   for (i = 0; i < niter; i += 1) {
       xmid = 0.5 * (x0 + x1);
       fmid = computeRd(xmid, twoC1, threeC2);
       if (fmid < Rd) {
           x0 = xmid;
       } else {
           x1 = xmid;
       }
   }

   return xmid;
}
```

Once α' is extracted from the albedo, the Better Dipoles can be summed up, considering a tuning diffuse mean free depth, as per the Skin BSSRDF notes ([Ren03]).

References

[D'E12] Eugene D'Eon. A better dipole. `http://www.eugenedeon.com/papers/betterdipole.pdf`, 2012.

[DI11] Eugene D'Eon and Geoffrey Irving. A quantized-diffusion model for rendering translucent materials. In *ACM SIGGRAPH 2011 papers*, SIGGRAPH '11, pages 56:1–56:14, New York, NY, USA, 2011. ACM.

[JMLH01] Henrik Wann Jensen, Stephen R. Marschner, Marc Levoy, and Pat Hanrahan. A practical model for subsurface light transport. In *Proceedings of ACM SIGGRAPH 2001*, Computer Graphics Proceedings, Annual Conference Series, pages 511–518, August 2001.

[Ren03] Renderman, theory and practice. `http://graphics.pixar.com/library/RMan2003/paper.pdf`, 2003.